

July 3, 2023

**BY EMAIL**

Mark Harper, Secretary of State for Transport  
United Kingdom  
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**Re: LED Headlamps – Response to DfT Letter of June 19, 2023**

Dear Mark Harper, Secretary of State for Transport, United Kingdom,

I am in receipt of the Department for Transport's letter dated June 19, 2023, where the DfT disagrees with our assertions that LED headlights require new regulations. While there are some areas of agreement, there are other areas where the DfT has made incorrect and invalid statements. The following paragraphs discuss the topic in detail.

**Definitions**

There are several important terms that the LED lighting industry has chosen to misuse. To engage in an honest discussion about LED light, we must first ensure that these terms are clearly defined.

**Point source** – A radiation source can be considered to be an infinitely small, mathematical point when the radiation source has curvature. As the mathematical point is inflated, the geometrical shape is a sphere and the radiation emitted by the surface of this sphere is uniform and disperses from the surface of the sphere. The stars and the sun can be modeled as point sources. While a candle has a cylindrical wick and a wax base, the physics and lighting terms used today are based on the candle being a point source. A tungsten filament is similarly considered to be a point source. A gas-discharge lamp is also a point source. All of these sources have curvature and an idealized spherical shape that can be considered as an infinitely small mathematical point.

**Flat surface source** – An LED chip has a flat surface. Thus, the geometry of an LED is different from that of a point source. The flat surface can never be modeled as a mathematical point which can be inflated into a sphere. No matter how small we make the chip and no matter how far away the viewer is from the chip, the chip will still maintain its geometrical shape of square or rectangle. The radiation emitted by a flat surface creates a directed energy beam, caused by overlapping light rays. The mathematical shape of this beam is called a Lambertian. A flat surface source can never be a point source and the physics formulas that were developed based on the candle and assuming a spherical point source are not necessarily valid flat surface sources. The radiation from a flat surface LED source

has a different energy at each point in 3D space, and thus simple formulas cannot be used. Instead, we must use calculus, integration, and 3D maps to describe the light energy from an LED.

Figure 1 shows a slide from an article by Professor Eijoon Yoon of Seoul National University.<sup>1</sup> In the slide, Professor Yoon categorizes the two types of emitters, point sources and surface sources. The brightness for a point source such as tungsten filament is measured using luminous intensity, while the brightness for a flat surface source such as LED is measured using luminance.

**Brightness and linearity of human vision**

- **Brightness: lack of standardized scientific definition**
  - **Brightness is an attribute of visual perception and is frequently used as synonym for luminance and (incorrectly) for the radiometric term radiance**
- **For point source,**
  - **Brightness (in the photopic vision regime) can be approximated by the luminous intensity (cd)**
- **For surface source,**
  - **Brightness can be approximated by luminance (cd/m<sup>2</sup>)**
- **Standard CIE**
  - **Assumption: human vision is linear within the photopic regime**
  - **Isotropically emitting blue point source and red point source have the same luminous intensity**

445.664 (Intro. LED) / Euijoon Yoon

Figure 1 - Point Source vs. Surface Source<sup>2</sup>

**Inverse Square Law** – The inverse square law states that the radiation from a point source dissipates following a  $1/r^2$  function.<sup>3</sup> The key is that this law is ONLY applicable to point sources. While the light from an LED does diverge in some way, because LEDs are not point sources, the light from an LED does not follow an inverse square law for dispersion and LED radiation maintains much of its density regardless of distance. Another example of a non-point source is a theoretically perfect laser which has collimated light that never diverges and thus does not follow the inverse square law. Formulas that rely on inverse square law dispersion cannot be used for LED flat surface sources because the divergence depends on the angle between the chip and the measurement location and due to the physical properties of the chip.

<sup>1</sup> <https://en.snu.ac.kr/research/highlights?md=v&bbsidx=121127>

<sup>2</sup> <https://ocw.snu.ac.kr/sites/default/files/NOTE/791.pdf>

<sup>3</sup> [https://en.wikipedia.org/wiki/Inverse-square\\_law](https://en.wikipedia.org/wiki/Inverse-square_law)

**Energy Efficiency** – Energy efficiency is providing the same quality of service using less energy.<sup>4</sup> The energy efficiency definition has two parts: a quality part and an energy part. For the quality part of the definition, there must first be a reference service. For the case of lighting, the reference service is often a tungsten filament light source. The tungsten filament provides the qualities of uniform illumination, inverse square law dispersion, a spectral power distribution of continuous increase from low blue to high red and infrared, and analog flicker. To make an energy efficiency comparison, the comparison product must produce these same qualities. If the comparison product does not provide these same qualities, then an energy efficiency comparison cannot be made, as the two products are simply different and provide different services.

An LED provides the qualities of non-uniform illumination, no inverse square law dispersion, a piecewise spectral power distribution frequently containing a large peak of blue wavelength, and square wave flicker. Since the qualities of an LED source are so drastically different than those of a tungsten filament source, no energy efficiency comparison can be made.

Despite these facts, there is a myth that only the luminous efficacy of two light sources is required to make an energy efficiency comparison or claim. This myth, while widely shared, is easily rebutted if we attempt compare the luminous efficacy of a laser and an incandescent light bulb. Which is more efficient? It's impossible to answer that question because the two products provide different services. Luminous efficacy can ONLY be used as the sole energy efficiency metric when the two light sources provide the same quality of service. The claim that LEDs are more energy efficient than a tungsten filament light cannot be made.

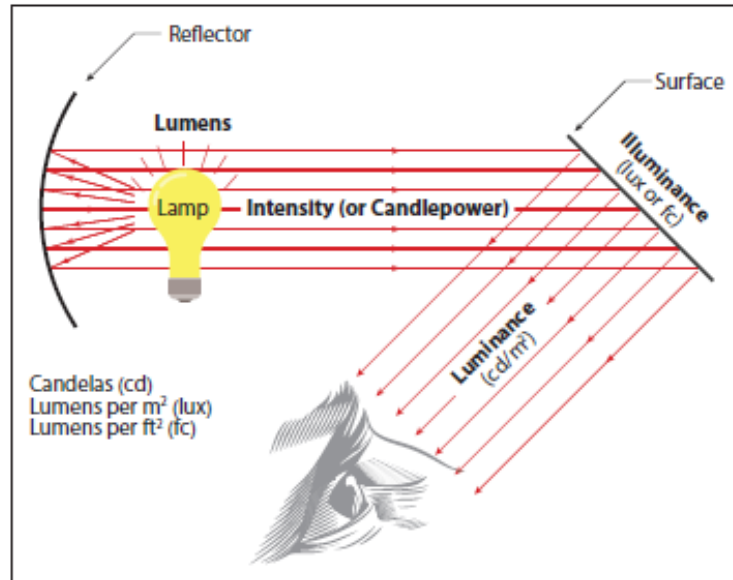
### Luminance

The lighting industry invalidly ignores the metric 'luminance'. While the LED chip industry brags about the ever-increasing peak luminance emitted by their chips, the lighting industry inexcusably ignores luminance and incorrectly attempts to describe the light emitted by flat surface LEDs using the same physics formulas and metrics that were developed for point sources.

Figure 2 from the Illuminating Engineering Society shows basic lighting terms. Notice in the diagram that the light is a point source. Thus, this diagram is not applicable to flat surface sources such as LEDs. The lighting industry standards are based on the metrics luminous flux (lumens), luminous intensity (candela) and illuminance (lux), while generally ignoring the metric luminance (candela per square meter). However, the diagram does correctly show that the luminance metric applies to a flat surface emitter.

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<sup>4</sup> <https://www.energy.gov/eere/energy-efficiency-buildings-and-industry>



**Relationship of basic lighting terms.**

*Figure 2 - Basic Lighting Terms<sup>5</sup>*

Figure 3 shows a photometry and radiometry comparison of the different lighting metrics.<sup>6</sup> Notice that 'luminance' is the correct metric for light emitted by a flat surface in a given direction. Thus, 'luminance' is the correct metric to use for regulating LED headlights, Daytime Running Lights, and other LED sources.

<sup>5</sup> Illuminating Engineering Society – RP-8-18

<sup>6</sup> <https://en.wikipedia.org/wiki/Luminance>

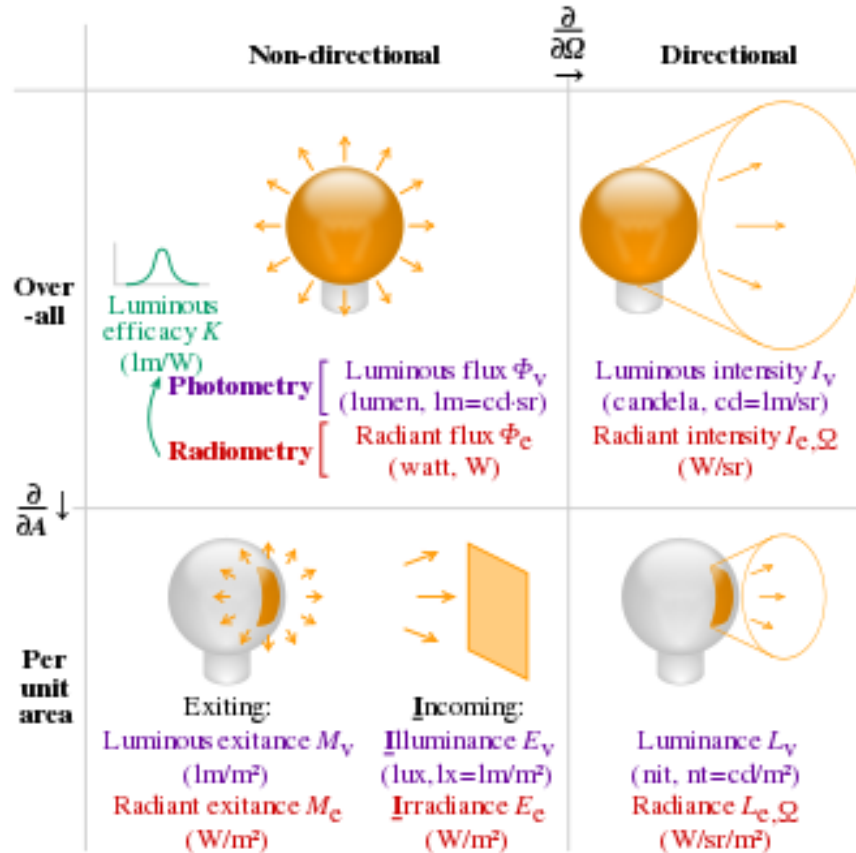


Figure 3 - The Luminance Metric

Luminance is measured in candela per square meter. Since a candela is a lumen per steradian, we can also state that luminance is measured in lumens per steradian per square meter. Luminance is the density of the light. For flat surface LEDs, there is not just one single luminance value. We can specify the 'peak luminance' which would be the luminance measured from the center of the chip where the most light rays overlap. We can also apply Lambert's cosine law to describe the luminance at some point in space in near field. What we cannot do is average or generalize the luminance to a single value, unless we specify the exact location in space that we are referring to.

Figure 4 shows the 2D intensity distribution of light emitted by an LED. The false colors represent the intensity at different angles normal to the flat surface of the chip. Notice that the most intense radiation is at center of the chip, falling off in intensity towards the edges. This radiation pattern is entirely different from the radiation emitted by a point source, which would show a single intensity at any given distance from the source. In addition, the spread in values between the light density at the edges and at the center is extreme when compared to human tolerances. The precision required to accurately measure the light density from a flat surface LED chip is in the femtometer or picometer range.

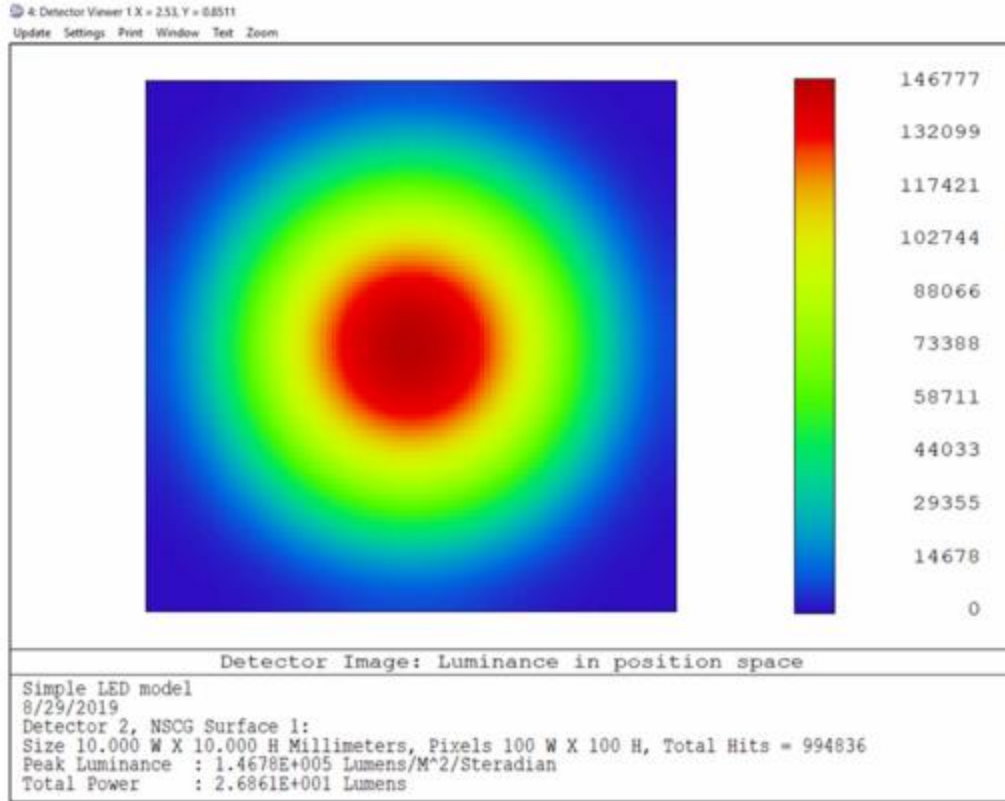


Figure 4 - LED Intensity Distribution<sup>7</sup>

Luminance is a weighted metric based on the sensitivity of an average human eye to light. The equivalent radiological term is radiance. Radiance is used to measure the energy density and power of the radiation. The power of the radiation when it reaches a human eye cell is what will determine the impact of radiation on that cell. A high-energy photon can cause instant cellular damage. Repeated exposure to lower levels of radiation can cause cell damage over time. There will be cumulative effects on the cell as it is exposed to radiation repeatedly. Eye cell damage is typically permanent.

The chart in Figure 5 shows some typical luminance values.<sup>8</sup> It must be made clear that luminance is the density of the light emitted or reflected by a flat surface. The row in the chart that says, “white paper lit by a candle at one foot” gives us a reference for the light source distance from the white paper, but after the reflection, the luminance value of 0.29 candela per square meter will remain the same, regardless of distance if the light rays are parallel. On the other hand, the luminance a 60-watt soft light (incandescent) bulb is listed as 10,000 cd/m<sup>2</sup>. The incandescent light bulb is a point source emitter with curvature, so the luminance measurement is made by a device placed immediately against the emitter to measure the density of the light from a flat surface at the source. This 10,000 cd/m<sup>2</sup> is not what is arriving at the eye at some distance because the point source nature of the emitter causes the light to disperse following an inverse square law. The luminance from a point source can only be measured at some distance after it reflects off a flat surface.

<sup>7</sup> <https://ieeexplore.ieee.org/stamp/stamp.jsp?arnumber=8879542>

<sup>8</sup> [https://www.atcorp.com/atcorp/media/pdfs/data-sheets/tektronix-j16\\_application.pdf](https://www.atcorp.com/atcorp/media/pdfs/data-sheets/tektronix-j16_application.pdf)

Light Source	Luminance, candela per square metre
Sun	$1.6 \times 10^9$
Arc lamp	$1.5 \times 10^8$
Metal halide lamp	$5.3 \times 10^6$
Clear incandescent lamp, filament	$2 \times 10^6$ to $2 \times 10^7$
Frosted incandescent lamp	50000 to 400000
Low pressure sodium lamp	75000
Maximum visual tolerance	50000
Cloud (sunny day)	35000
Fluorescent lamp	12000 to 14000
White illuminated cloud	10000
60 watt soft-white bulb	10000
Surface of moon	1000
Metal-halide flood lamp	500
Convenience store sign	150
White paper under lamp	30 to 50
Television screen (CRT)	9
Neon lamp	8
Candle	7.5
Clear sky	3 to 5
Moon	2.5
White paper lit by candle at one foot	0.29
Dark sky reserve (proposed)	0.1
Night sky	0.001
Threshold of vision	0.000003

Figure 5 - Typical Luminance Values

The table in Figure 5 gives a sense of what our eyes, nerves, and brains are naturally adapted to. We have all been taught to not look at the sun due to risk of eye damage. The luminance from the sun is 1,600,000,000 candela per square meter. An arc lamp is 150,000,000 cd/m<sup>2</sup>. At the other end of the scale is the threshold of vision at 0.00003 cd/m<sup>2</sup>. Our vision system still functions at this extremely low level of luminance. Most people are comfortable looking at a candle or a neon sign, both of which have a luminance of approximately 8 cd/m<sup>2</sup>.

What value in Figure 5 is the upper limit for human comfort and the upper limit for human tolerance? Since LEDs are flat source emitters, regulations must limit the peak luminance emitted by the LED. For example, Section 11.2.2 Visible and infra-red collateral radiation in IEC 60825-1 states, "The visible and near infra-red radiation emitted from flash tubes and pump sources and target re-radiation may be of sufficient radiance to produce potential hazard."<sup>9</sup> What is the peak radiance and/or

<sup>9</sup> [https://shop.textalk.se/shop/ws26/40626/files/full\\_size\\_-\\_for\\_start\\_page\\_banner/iec60825-1%7Bed1.2%7Den.pdf](https://shop.textalk.se/shop/ws26/40626/files/full_size_-_for_start_page_banner/iec60825-1%7Bed1.2%7Den.pdf)

luminance from an LED source that causes discomfort, pain, eye injury, or neurological trauma such as seizures, migraines, panic attacks or interference with cognitive functioning?

The chart shown in Figure 6 was published by the LED headlight manufacturer Hella. The far-right column header is “Luminance” in Megacandela per square meter. For an LED headlight as of 2013, the peak luminance is listed as 70,000,000 candela per square meter.

### 3 LED HEADLIGHTS ADVANTAGES: COMPARISON

LEDs are superior in several aspects. They might be more expensive to purchase than normal light bulbs or halogen bulbs, but their use pays for itself in a short time. The automotive industry in particular uses the positive features of the LED and employs it increasingly in new vehicles due to the following advantages:

Light Source	Luminous flux [lm]	Efficiency [lm/W]	Colour temperature [K]	Luminance [Mcd/m <sup>2</sup> ]
Conventional bulb W5W	~ 50	~ 8	~ 2700	~ 5
Halogen bulb H7	~ 1100	~ 25	~ 3200	~ 30
Gas discharge D2S	~ 3200	~ 90	~ 4000	~ 90
LED 2.5 Watts	~ 120 (2010) ~ 175 (2013)	~ 50 (2010) ~ 70 (2013)	~ 6500	~ 45 (2010) ~ 70 (2013)

Figure 6 - Headlight Luminance<sup>10</sup>

Is this level of luminance (70,000,000+ cd/m<sup>2</sup>) comfortable or safe, both for the eye and for the eye/nerve/brain vision system? Figure 7 shows the levels of luminance for comfort (8 cd/m<sup>2</sup>), maximum comfort limit (300 cd/m<sup>2</sup>)<sup>11</sup> and an LED headlight (70,000,000 cd/m<sup>2</sup>). **Considering that the peak luminance of an LED headlight is in the same range as an arc lamp, it should be immediately obvious that an LED headlight is hazardous and dangerous.**

<sup>10</sup> Hella - <https://www.hella.com/techworld/us/Technical/Automotive-lighting/LED-headlights-833/>

<sup>11</sup> <https://www.softlights.org/wp-content/uploads/2022/10/DigitalBillboardsIanLewin-highlighted.pdf>



	Light Source	Luminance, candela per square metre	
LED headlight	Sun	$1.6 \times 10^9$	
	Arc lamp	$1.5 \times 10^8$	
	Metal halide lamp	$5.3 \times 10^6$	
	Clear incandescent lamp, filament	$2 \times 10^6$ to $2 \times 10^7$	
	Frosted incandescent lamp	50000 to 400000	
	Low pressure sodium lamp	75000	
	Maximum visual tolerance	50000	
	Cloud (sunny day)	35000	
	Fluorescent lamp	12000 to 14000	
	White illuminated cloud	10000	
	60 watt soft-white bulb	10000	
	Surface of moon	1000	
	Comfort Limit	Metal-halide flood lamp	500
		Convenience store sign	150
Comfortable	White paper under lamp	30 to 50	
	Television screen (CRT)	9	
	Neon lamp	8	
	Candle	7.5	
	Clear sky	3 to 5	
	Moon	2.5	
	White paper lit by candle at one foot	0.29	
	Dark sky reserve (proposed)	0.1	
	Night sky	0.001	
	Threshold of vision	0.000003	

Figure 7 - Luminance Limits

### Spatial Shape of LED Light

It is critical to note that there are at least two categories of emitters. Not all emitters can be considered point sources. Examples of point sources include the sun, a candle, tungsten filament, and gas-discharge lamps. While none of these sources are perfectly spherical, generally they can be modeled as a spherical point source, with the light radiating outwards following an inverse square law for energy dispersion. Flat surface sources, on the other hand, include lasers and LEDs. The surface of the emitter has no curvature, and thus the light is emitted as a directed energy beam due to the overlapping light rays. In the case of a theoretically perfect laser, the light is collimated, and never disperses, maintaining its density to infinity. In the case of LEDs, the mathematical shape of the directed energy beam is a Lambertian, with the peak luminance being the center of a square LED chip, falling off

in energy density towards the edges of the chip. There is nearly zero energy emitted behind the flat chip.

Notice in the radiation profile shown in Figure 8 that the peak intensity for an LED emitter is at zero degrees and that the intensity falls off as the angle approaches 90 degrees. The chart shows relative intensity, but we know from the Hella table that for a vehicle headlight, the peak intensity is at least 70,000,000 cd/m<sup>2</sup>.

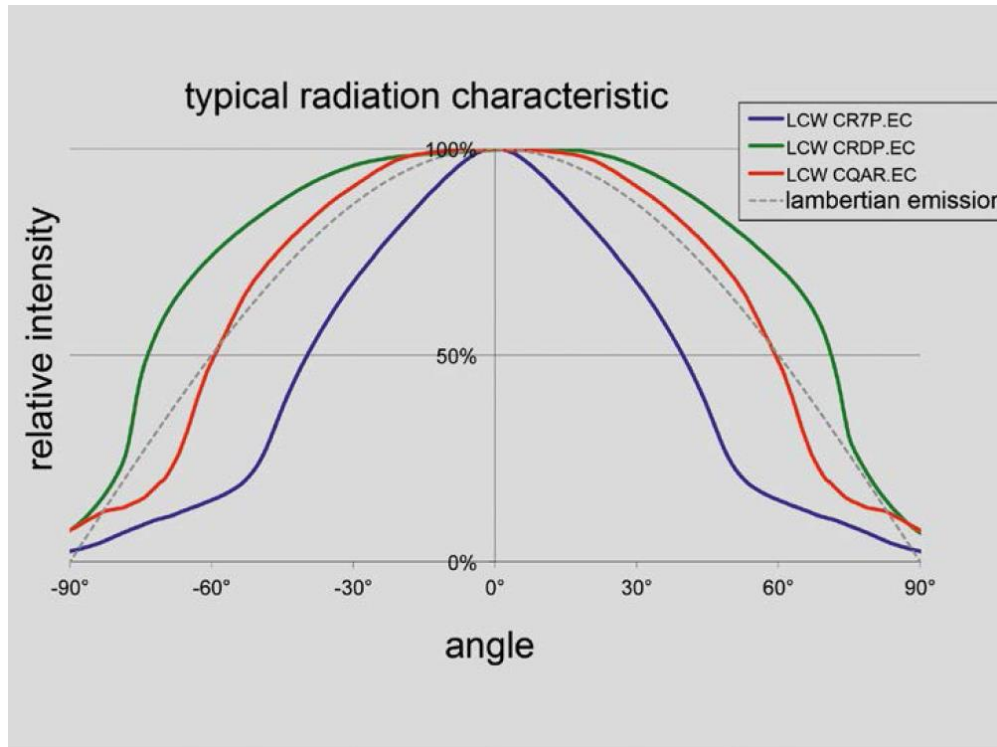


Figure 8 - LED Relative Intensity

This radiation intensity profile for a flat surface emitter is completely different than for a point source emitter. For a point source, the emitter is curved and the emitted radiation is generally homogenous, and disperses gently due to the curvature of the source. For a tungsten filament, the emitter is more like a cylinder, so the density of the light will not be perfectly equal at all locations in space, but the differences between the highest intensity and the lowest intensity will be small, and as the light radiates outward and the density decreases following an inverse square law, the differences will be even smaller.

On the other hand, the light from a flat surface LED is extremely dense at the center of the chip, and because the light energy is a directed energy beam caused by the overlapping light rays emitted by the flat chip, there is very little divergence as the distance varies. It cannot be said that the light from an LED chip follows an inverse square law for dispersion, because every point in space is a different energy. There is no method for applying a simple  $1/r^2$  law to radiation that has entirely different energies at each point in 3D space. Even the entry in Wikipedia states, *“The inverse-square law generally applies when some force, energy, or other conserved quantity is evenly radiated outward from a point source in three-dimensional space. Since the surface area of a sphere (which is  $4\pi r^2$ ) is proportional to the square*

of the radius, as the emitted radiation gets farther from the source, it is spread out over an area that is increasing in proportion to the square of the distance from the source.”<sup>12</sup> Thus, ONLY if the radiation is emitted by a point source can the inverse square law be applied. Since we have already shown that LED emitters are not point sources, it is not possible to claim that the radiation emitted by an LED follows an inverse square law for dispersion.

A curved surface emitter such as a tungsten filament will emit essentially spatially uniform isotropic radiation as shown in (a) and (c) of Figure 9. A flat surface emitter such as an LED will emit spatially non-uniform anisotropic radiation, as shown in (b) and (d).

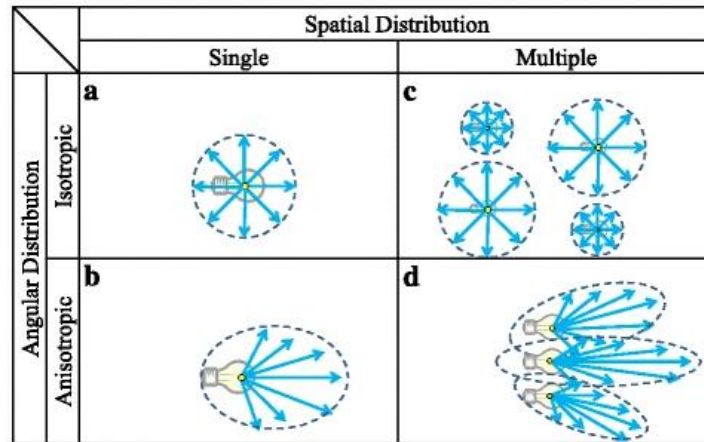


Figure 9 - Isotropic vs. Anisotropic<sup>13</sup>

This is why the public is complaining of eye pain and eye injury when exposed to LED headlights. First, the density is exceedingly high at 70,00,000+ candela per square meter, and second, there is little loss of energy density from an LED beam as the light propagates through space. There are currently no known governmental limits on peak luminance or a requirement that the light disperse gently following an inverse square law.

#### Department for Transport Quotes

The Department for Transport article Technical Annex – LED light sources in automotive applications contains errors that must be corrected and addressed.

Quote 1: “*The regulations are correct for historical light sources and remain correct for LEDs.*” – As explained in the paragraphs above, there must be two sets of regulations, one for point sources and a different set for flat surface sources, because of the radically different physics characteristics of the two source types.

Quote 2: “*...the primary metric for categorising headlamp bulbs was in lumens. This has since been adapted such that headlamp performance is now measured by Intensity (lm/steradian)...*” – It is

<sup>12</sup> [https://en.wikipedia.org/wiki/Inverse-square\\_law](https://en.wikipedia.org/wiki/Inverse-square_law)

<sup>13</sup> <https://ipsjcvva.springeropen.com/articles/10.1186/s41074-016-0014-z>

time once again to update the metrics for measuring intensity from headlamps. The correct metric for flat surface LED emitters is luminance (lumens/steradian/meter<sup>2</sup>).

Quote 3: *“A lamp designer is primarily interested in the source lumens from the LEDs/bulbs etc to determine how many light sources they require to sufficiently illuminate the road scene.”* – Because of the extreme luminance of LED lamps, the primary focus of the lamp designer must be protecting the eyes and neurological functioning of those people on the receiving end of the light.

Quote 4: *“Luminance is not the correct metric to use when determining the light source requirements to sufficiently illuminate a road scene.”* – As detailed in the paragraphs above, for flat surface LED sources, luminance is indeed the correct metric.

Quote 5: *“...for example, a 500 nit TV screen...”* – This is a good example that shows that LED light sources are measured with luminance (cd/m<sup>2</sup> or nit). Luminance is the correct metric for intensity from a flat surface source. As shown in the chart above, a 500 nit TV exceeds the maximum comfort level for many people. What is the impact of a 70,000,000 nit LED headlight?

Quote 6: *“Should an artificially low luminance limit be imposed as the metric for measurement, based upon incorrect information, it will drive the size and weight of headlamps to increase.”* – This is a technology-focused statement, rather than a human-focused statement. The Department for Transport is prioritizing technical achievement over human health.

Quote 7: *“It is incorrect to suggest that the light emitted from an LED does not follow an inverse square for dispersion. The Inverse-Square law applies to any wavefront of light propagating through free space regardless of the type of source.”* – As detailed in the paragraphs above, this statement is incorrect. The inverse square law can only be applied to point sources, not flat surface sources.

Quote 8: *“A standard 90mm diameter aspheric lens, as used in many headlamps, has an area of 0.0064m<sup>2</sup>, 300 nits (cd/m<sup>2</sup>) from 0.0064m<sup>2</sup> is equivalent to 1.9 cd.”* – This is fundamentally flawed mathematics. Stating that the luminance metric is equivalent to the luminous intensity metric is like saying that 1 meter is equivalent to 5 grams.

Quote 9: *“United Nations ECE Regulation 149 requires headlamp beam patterns to have a minimum of 12100cd at the 75R test point.”* – As detailed in the paragraphs above, this regulation is only applicable to point source emitters. For flat surface LED emitters, the test would need to measure luminance.

Quote 10: *“Meanwhile in the direction of the B50L test point (oncoming drivers eye point in UNECE regulations), the regulations allow for a maximum of 350cd, or an equivalent surface luminance of approximately 55,000 nits.”* – Ignoring for a moment whether the math conversion from 350cd to 55,000 nits is correct, the Department of Transportation is stating that the limit for a vehicle headlight is 55,000 cd/m<sup>2</sup>, and yet Hella has stated that LED headlights are 70,000,000 cd/m<sup>2</sup>. This statement appears to confirm that LED headlights exceed existing regulations and that vehicles using LED headlights must be recalled.

Quote 11: *“There is nothing inherently unsafe about having a light distribution that is inhomogeneous provided it is controlled to avoid putting too much illuminance in the direction of oncoming drivers.”* – The collected epidemiological data shows clearly that high-luminance LED lamps cause seizures, migraines, panic attacks, and eye injury. The Lambertian distribution of the directed

energy beam may indeed be one of the causes of these adverse neurological impacts because the human nervous system is not adapted to the extreme density changes from 70,000,000 nits to 0 nits distributed on the eye simultaneously.

Quote 12: *“Again, the point is that the geometry of the source is not important when it comes to the safe dispersion characteristics of headlamps only that the emitted beam pattern from the complete headlamp should be considered.”* – The information presented in this paper makes clear that the geometry of the source is fundamental.

Quote 13: *“The wavefront that is observed by road users bears no relation to the emission distribution of the source LED.”* – The street light company Cree published the following statement: *“None of the existing metrics takes into account the non-uniform emitting surface of a LED luminaire.”*<sup>14</sup> We are unaware of any technology in use today in LED headlamp systems that disperses the spatially non-uniform light such that the original 70,000,000 nit density is reduced to a humanly comfortable 300 nits. The primary reason why road users have complained so bitterly about LED headlights is exactly because the emission from the source LED is excessively intense and unregulated.

Quote 14: *“We would be interested to see evidence showing that inhomogeneous light distributions can cause neurological harm, all else being equal, photons are photons, regardless of what type of source emits them.”* – As noted previously, there is an abundant collection of incident reports from the public detailing the seizures, migraines, and panic attacks induced by LED lighting.<sup>15</sup> The idea that any given set of spatial, spectral, and temporal properties of photons can have no neurological impact is not justifiable.

Quote 15: *“The majority of automotive LEDs fall under Risk Groups 0 & 1 for which no damage will occur during exposure times of up to 100s.”* – There is no justification for this premise.

Quote 16: *“Some blue LEDs and the higher power white LEDs do fall into Risk Group 2 for which damage may occur for exposure times of up to 0.25s however, due to the ‘aversion response’ to bright light, no hazard is likely.”* – In an example with 100 vehicles on a road headed home from work in rush hour traffic, the Department for Transport is suggesting that each driver will suffer a pain incident from the oncoming vehicle, close their eyes or turn their head, spend the next 20 seconds attempting to recover their vision, and then repeat the process. As seen earlier, the Department for Transport is presenting a technology-centric view, allowing human comfort and safety to be sacrificed for the benefit of the technology, rather than putting the technology to work to benefit humans.

Sincerely,

/s/ Mark Baker

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<sup>14</sup> <http://www.softlights.org/wp-content/uploads/2022/04/Cree-Lighting-White-Paper.pdf>

<sup>15</sup> <http://www.softlights.org/stories/>

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8. Healthier and Environmentally Responsible Sustainable Cities and Communities. A New Design Framework and Planning Approach for Urban Illumination - <https://www.mdpi.com/2071-1050/14/21/14525/htm> - Artificial light is having significant negative consequences on human and biological health. Over 100 references to research studies.
9. The Influence of a Photometric Distance on Luminance Measurements - <https://www.mdpi.com/1996-1073/16/10/4166> - Discussion of how to measure luminance from an LED. The author measured a peak luminance of 13,000,000 candela per square meter from an LED chip.